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AN ECONOMIC ORDER QUANTITY MODEL WITH VARIABLE DEMAND PATTERN, WEIBULL DETERIORATION AND SHORTAGES IN PARTIAL BACKLOGGING ENVIRONMENT

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ABSTRACT:
In present paper, an inventory model has been developed for perishable products. Three parameter Weibull distributions are used to model deterioration. The demand is considered to be combination of linear function and constant function of time. Shortages are also incorporated allowed and partially backlogged. Backlogging rate is considered to dependent on waiting time. Proposed model is illustrated with support of numerical example. Sensitivity of model is also analyzed with respect to different parameters.
Keywords: Inventory, Deterioration, Partial backlogging, Variable demand

INTRODUCTION

Inventory control is a key aspect in any manufacturing and distribution system. Demand pattern plays an important role in deciding the optimal inventory level. Research is carried out with different type of demand patterns. In some models, demand of the items is assumed to be constant. In other, it is taken to be increasing linearly. It is also considered to be exponential with respect to time in some models. Inventory models with time-dependent demand were studied by Maiti et al. (2009).
Mandal and Pal (1998) have suggested inventory models with ramp type demand rate for perishable items. Panda et al. (2008) have suggested optimal replenishment model for deteriorating products with short product cycle. Avinadav et al. (2013) have considered demand function which is sensitive with respect to both price and time. Models for seasonal products with ramp-type time-dependent demand are discussed by Wang and Huang (2014). Bhunia et al. (2017) have proposed a model with variable demand and flexible reliability.

Deterioration is defined as decay in quality of product with passage of time. Most of the consumable products are subjected to deterioration in quality or become completely useless with passage of time. Modelers have also considered this factor in their models. Different types of order-level inventory models for items deteriorating at a constant rate were discussed by Shah and Jaiswal (1977) and Dave (1986). Several researchers have developed inventory models with deterioration with variable rate. Covert and Philip (1973), Chakrabarti et al. (1998), Jalan et al. (1996)and Dye (2004) have significant contribution in this regard. Manna and Chaudhuri (2006) have proposed an EOQ model with ramp type demand rate and deterioration rate to be function of time. An inventory system with Markovian demands, phase type distributions for perishability and replenishment is developed by Chakravarthy (2011). San-José et al. (2014) have studied inventory system with partial backlogging and mixture of dispatching policies. Wu et al.(2016) have studied time-dependent deterioration with trapezoidal-type demand.

When there are shortages for a product, some demand is lost while some customers wait for next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. This phenomenon in which some demand is lost and remaining part of the demand is fulfilled in next replenishment is called partial backlogging.

Chang and Dye (1999) have suggested an inventory model in which the shortages are partially backlogged. Papachristos and Skouri (2000) have considered time-dependent partial backlogging in their model. Teng et al. (2003) further extended this model by assuming backlogged demand to any decreasing function of the waiting time up to the next replenishment. The work by Teng and Yang (2004), Dye et al. (2006), Singh and Singh (2007, 2009) etc. can also be consulted in this connection. San-Jose et al. (2015) have studied partial backlogging with non linear holding cost. Chand et al. (2016) have suggested a periodic review inventory model with period-dependent backlogging costs.

In the present paper, we have studied an EOQ model for time-dependent deteriorating items assuming the demand rate to be a piecewise function of time. Such demand has been modeled by the use of Heaviside's unit step function. The demand rate for such items increases with time up to certain time and then ultimately stabilizes and becomes constant. It is believed that such type of demand rate is quite realistic.

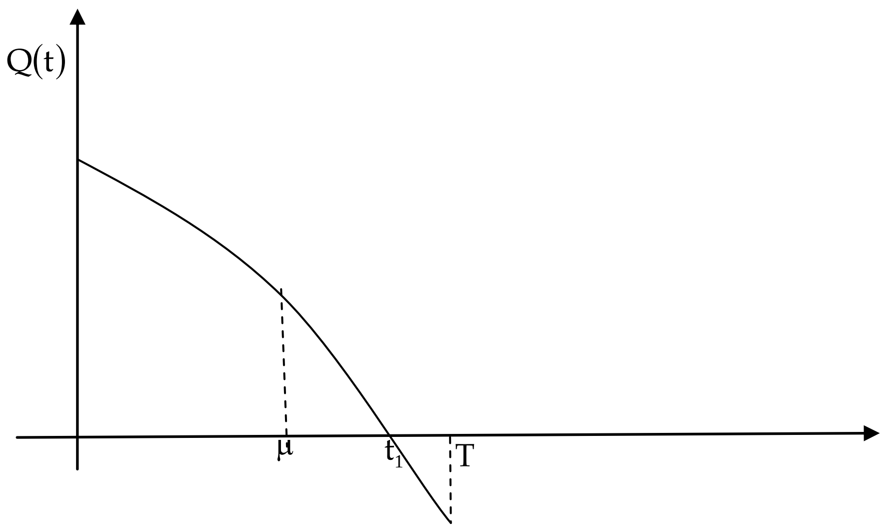
ASSUMPTIONS AND NOTATIONS:

In the proposed model, the following notations and assumptions are used:

i) There is no delay between placing an order and arrival of items.
ii) c_1 is the inventory holding cost per unit per unit of time.
iii) c_3 is the shortage cost per unit per unit of time.
iv) c_4 is the unit cost of lost sales.
v) c_5 is the cost of each unit.
vi) Ordering cost is $c.'$
vii) Demand rate is a combination a linear and constant function of time defined by $f(t) = a\{t - (t-\mu)H(t-\mu)\}$ Where a & μ are constants and $H(t-\mu)$ is Heviside's function defined as follows: $H(t-\mu) = \begin{cases} 0, t < \mu \\ 1, t \geq \mu \end{cases}$
viii) Unsatisfied demand is backlogged at a rate $e^{-\lambda t}$, where t is the time up to next replenishment and λ is a positive constant.
ix) R is the total cost per production cycle and T is the time for each cycle.
x) $Q(t)$ be the inventory level at time t.
xi) The distribution of the time to deterioration of the items follows three parameter Weibull distributions. Thus a variable fraction $\theta(t) = \alpha(t-\gamma)^{\beta-1}$, $(0 < \alpha \ll 1, t \geq 0)$ is the deterioration rate.

FORMULATION AND SOLUTION OF THE MODEL:

At the beginning of the cycle, the inventory level reaches its maximum S at time t=0. During the time interval$[0, t_1]$, the inventory level decreases due to demand and deterioration. At time $t = \mu < t_1$, the inventory level decreases and at t_1, the inventory level is zero and all the demand hereafter (i.e. $T - t_1$) is partially backlogged. The demand varies with time up to a certain time and become constant thereafter. The deterioration rate is described by an increasing function of time $\theta(t) = \alpha(t-\gamma)^{\beta-1}$.
A Graphical representation of the considered inventory system is given below:



The differential equations governing the instantaneous states of $Q(t)$ in the interval $[0,T]$ are as follows:

$$\frac{dQ(t)}{dt}+\theta(t)Q(t)=-f(t), 0\le t\le\mu \qquad (1)$$

$$\frac{dQ}{dt}+\theta(t)Q(t)=-f(t), \mu\le t\le t_1 \qquad (2)$$

$$\frac{dQ}{dt}=-f(t)\,e^{-\lambda t}\;, t_1\le t\le T \qquad (3)$$

Conditions are $Q(0)=S$, $Q(t_1)=0$

The solutions of equations (1) to (3) are given below:

$$Q(t)=e^{-\alpha(t-\gamma)^{\beta}}\left[e^{\alpha(-\gamma)^{\beta}}S+\frac{a\alpha(-\gamma)^{1+\beta}\gamma}{(1+\beta)(2+\beta)}-a\left\{\frac{t^2}{2}+\frac{\alpha(t-\gamma)^{1+\beta}(t+t\beta+\gamma)}{(1+\beta)(2+\beta)}\right\}\right],\ 0\le\ t\le\mu \quad (4)$$

$$Q(t)=-\frac{a e^{-\alpha(t-\gamma)^{\beta}}\alpha\mu(\gamma-t_1)(-\gamma+t_1)^{\beta}}{1+\beta}-\frac{a e^{-\alpha(t-\gamma)^{\beta}}\mu\{t+t\beta+t\alpha(t-\gamma)^{\beta}-\alpha(t-\gamma)^{\beta}\gamma-t_1-\beta t_1\}}{1+\beta},\ \mu\le t\le t_1 \quad (5)$$

$$Q(t)=\frac{a\left(e^{-t\lambda}-e^{-\lambda t_1}\right)\mu}{\lambda},\ t_1\le t\le T \qquad (6)$$

Using above relations, S is given by

$$S=\frac{1}{2(1+\beta)(2+\beta)}a e^{-\alpha(-\gamma)^{\beta}}\left[(1+\beta)(2+\beta)\mu(-\mu+2t_1)+2\alpha\{-(-\gamma+\mu)^{2+\beta}+(2+\right.$$

$$\beta)\mu t1-\gamma+t1\beta+-\gamma\beta\gamma2-2+\beta\mu\gamma(-\gamma+t1)\beta \qquad (7)$$

The inventory holding cost during the interval $(0,T)$ is given by

$$C_H=c_1\left[\int_0^{\mu}Q(t)dt+\int_{\mu}^{t_1}Q(t)\,dt\right]$$

$=\frac{c_1}{6(1+\beta)(2+\beta)(3+\beta)}\left[6e^{\alpha(-\gamma)^{\beta}}S(2+\beta)(3+\beta)\{\alpha(-\gamma)^{1+\beta}+\mu+\beta\mu-\alpha(-\gamma+\mu)^{1+\beta}\}+\right.$

$a-(1+\beta)(2+\beta)(3+\beta)\mu 3+6a-\gamma 2+\beta 3\gamma-3+\beta\mu+3a-\gamma+\mu 1+\beta 6\gamma 2+4\beta\gamma\mu+\beta(1+\beta)\mu 2$

$+c_1 a\mu\left[\frac{\mu^2}{2}-\frac{2\gamma(-\gamma+\mu)^{1+\beta}}{2+3\beta+\beta^2}-\frac{\beta\mu(-\gamma+\mu)^{1+\beta}}{2+3\beta+\beta^2}-\mu t_1+\frac{2(-\gamma+\mu)^{1+\beta}t_1}{2+3\beta+\beta^2}+\frac{\beta(-\gamma+\mu)^{1+\beta}t_1}{2+3\beta+\beta^2}+\frac{t_1^2}{2}-\frac{\mu(-\gamma+t_1)^{1+\beta}}{1+\beta}+\right.$

$t1(-\gamma+t1)1+\beta 1+\beta-2(-\gamma+t1)2+\beta 2+3\beta+\beta 2$

(8)

The cost due to deterioration of units in the period $(0,T)$ is given by

$C_D = c_5$ (Initial inventory level-Total units sold)

$$= c_5\left[S-\int_0^{t_1} f(t)\,dt\right]$$

$= c_5\left[-\frac{a\mu^2}{2}-a\mu(-\mu+t_1)+\frac{1}{2(1+\beta)(2+\beta)}ae^{-\alpha(-\gamma)^{\beta}}\{(1+\beta)(2+\beta)\mu(-\mu+2t_1)+\right.$

$2a--\gamma+\mu 2+\beta+2+\beta\mu t1-\gamma+t1\beta+-\gamma\beta\gamma 2-2+\beta\mu\gamma-\gamma+t1\beta$ (9)

The cost due to shortages in the interval $(0,T)$ is given by

$$C_S = -c_3\left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$= -c_3\left(-\frac{ae^{-T\lambda}\mu}{\lambda^2}+\frac{ae^{-\lambda t_1}\mu}{\lambda^2}-\frac{aTe^{-\lambda t_1}\mu}{\lambda}+\frac{ae^{-\lambda t_1}\mu t_1}{\lambda}\right) \qquad (10)$$

The opportunity cost due to lost sales in the interval $(0,T)$ is given by

$C_O = c_4\left[\int_{t_1}^{T}(1-e^{-\lambda t})f(t)dt\right]$

$$= a\mu c_4\left(T+\frac{e^{-T\lambda}-e^{-\lambda t_1}}{\lambda}-t_1\right) \qquad (11)$$

The total cost R in the system in the interval $(0,T)$ is given by

$$R = c' + C_H + C_D + C_S + C_O \qquad (12)$$

In above relation, c' is constant, while $C_H, C_D, C_S \& C_O$ are given by the equations (8) to (11).

The average cost K in the system in the interval $(0,T)$ is given by

$$K = \frac{R}{T} \qquad (13)$$

The optimum values of t_1 and T which minimize average cost K are obtained by using the equations:

$$\frac{\partial K}{\partial t_1} = 0 \text{ and } \frac{\partial K}{\partial T} = 0\,,$$

Now,

$$\frac{\partial K}{\partial t_1} = 0$$

⇒

$$a\left(-1+e^{-\lambda t_1}\right)\mu c_4 - a\mu c_5 - a e^{-\lambda t_1}\mu c_3 (T-t_1) +$$

$$\frac{a\mu c_1\left[-\mu-\beta\mu-\gamma(-\gamma+\mu)^{\beta}+\mu(-\gamma+\mu)^{\beta}+t_1+\beta t_1+(-\gamma+t_1)^{\beta}\{\gamma-(1+\beta)\mu+\beta t_1\}\right]}{1+\beta}=0 \qquad (14)$$

Also, $\frac{\partial K}{\partial T}=0$ gives

$$c' + \frac{aT\left(e^{-T\lambda}-e^{-\lambda t_1}\right)\mu c_3}{\lambda} - Ta\left(1-e^{-T\lambda}\right)\mu c_4 + a\mu c_4\left(T+\frac{e^{-T\lambda}-e^{-\lambda t_1}}{\lambda}-t_1\right) - c_3\left(-\frac{ae^{-T\lambda}\mu}{\lambda^2}+\frac{ae^{-\lambda t_1}\mu}{\lambda^2}-\right.$$

aTe− λt1μλ+ae− λt1μt1λ

$$+c_1\left[\frac{1}{6(1+\beta)(2+\beta)(3+\beta)}\left\{6e^{\alpha(-\gamma)^{\beta}}S(2+\beta)(3+\beta)\left(\alpha(-\gamma)^{1+\beta}+\mu+\beta\mu-\alpha(-\gamma+\mu)^{1+\beta}\right)+\right.\right.$$

α− 1− β2+β3+βμ3+6α− γ2+β3γ− 3+βμ+3α− γ+μ1+β6γ2+4βγμ+β1+βμ2+aμμ22− 2γ(− γ+μ
)1+β2+3β+β2− βμ(− γ+μ)1+β2+3β+β2− μt1+2(− γ+μ)1+βt12+3β+β2+β(− γ+μ)1+βt12
+3β+β2+t122− μ(− γ+t1)1+β1+β+t1(− γ+t1)1+β1+β− 2(− γ+t1)2+β2+3β+β2

$$+c_5\left[-\frac{a\mu^2}{2}-a\mu(-\mu+t_1)+\frac{1}{2(1+\beta)(2+\beta)}ae^{-\alpha(-\gamma)^{\beta}}\left\{(1+\beta)(2+\beta)\mu(-\mu+2t_1)+2\alpha\left(-(-\gamma+\right.\right.\right.$$

μ)2+β+(2+β)μt1(− γ+t1)β+γ((− γ)βγ−(2+β)μ(− γ+t1)β) (15)

NUMERICAL EXAMPLE:

To illustrate the model numerically, we use the following parameter values:

$c_1=2.6,\ c_3=4,\ c_4=12,\ c_5=7,\ c'=110, \mu=0.6,\ \alpha=0.003,\ \beta=25,\ \gamma=0.5,\ a=8500,$
$\lambda=0.12$

Applying the subroutine FindRoot in Mathematica 8, we obtain the optimal solution for t_1 and T as follows:

$$t_1=1.56246, T=1.67521$$

Also, the optimal average cost for these parameters is 11607.9

SENSITIVITY ANALYSIS:

Sensitivity analysis is performed by changing (increasing and decreasing) the parameters by 10%, 30% and 50%, and taking one parameter at a time, keeping the remaining parameters at their original values. Thus following table is formed:

Table1

Changing Parameter	**% Change**	t_1	T	S	**Average Cost**
c_1	+50	1.539	1.814	7737	16344
	+30	1.548	1.759	7787	14511
	+10	1.557	1.704	7844	12598
	−10	1.568	1.647	7912	10594
	−30	1.582	1.610	7996	8486
c'	+50	1.562	1.676	7877	11637
	+30	1.562	1.676	7877	11625
	+10	1.562	1.675	7877	11614
	−10	1.562	1.675	7877	11614
	−30	1.562	1.675	7877	11590
	−50	1.562	1.674	7877	11579
c_3	+50	1.563	1.643	7878	11648
	+30	1.563	1.653	7878	11635
	+10	1.563	1.667	7877	11618
	−10	1.562	1.685	7876	11596
	−30	1.562	1.711	7875	11565
	−50	1.562	1.751	7874	11517
c_4	+30	1.563	1.588	7880	11737
	+10	1.563	1.645	7878	11666
	−10	1.562	1.706	7876	11535
	−30	1.562	1.768	7874	11344
	−50	1.562	1.834	7873	11092
c_5	+50	1.584	1.722	8009	12454
	+30	1.577	1.704	7962	12123
	+10	1.568	1.685	7908	11783
	−10	1.557	1.665	7843	11429
	−30	1.543	1.641	7761	11056
	−50	1.525	1.612	7650	10649
α	+50	1.562	1.675	7878	11614
	+30	1.562	1.675	7877	11611
	+10	1.562	1.675	7877	11609
	−10	1.562	1.675	7877	11607
	−30	1.562	1.675	7876	11604
	−50	1.562	1.675	7876	11602
β	+50	1.541	1.639	7749	11048
	+30	1.548	1.650	7788	11222
	+10	1.557	1.665	7842	11457
	−10	1.570	1.687	7920	11791
	−30	1.590	1.722	8045	12303
	−50	1.629	1.784	8274	13192
γ	+50	1.846	1.976	9574	13453
	+30	1.732	1.855	8894	12712

	+10	1.619	1.735	8215	11973
	−10	1.506	1.616	7540	11246
	−30	1.395	1.498	6870	10535
	−50	1.284	1.383	6206	9841
μ	+50	1.588	1.681	10600	17197
	+30	1.577	1.679	9604	15056
	+10	1.567	1.677	8485	12785
	−10	1.558	1.674	7235	10407
	−30	1.550	1.670	5850	7946
	−50	1.542	1.666	4327	5427
λ	+50	1.563	1.587	7882	11751
	+30	1.563	1.598	7880	11729
	+10	1.563	1.649	7878	11660
	−10	1.562	1.701	7876	11543
	−30	1.562	1.753	7874	11378
	−50	1.562	1.806	7873	11162

From Table 1, the following points are noted:
(i) It is seen that the percentage change in the optimal cost is almost equal for both positive and negative changes of all the parameters exceptc_4 , β and μ
(ii) It is observed that the model is more sensitive for a negative change than an equal positive change in the parameter c_4, μ & β.
(iii) The optimal cost increases (decreases) and decreases (increases) with the increase (decrease) and decrease (increase) in the value of the parameters $c_1, c', c_3, c_4, c_5, \alpha, \gamma, \mu,$ & λ but this trend is reversed for the parameterβ.
(iv) Model is highly sensitive to changes in c_1, μ & γ and moderately sensitive to changes in c_5 & β . It has low sensitivity to c', c_3, c_4, α & λ.
(v) From the above points, it is clear that much care is to be taken to estimate c_1, μ & γ.

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